ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-II

Paper Code: C4T

[Differential Equations and Vector Calculus]

Answer all the questions

1. Define fundamental set of solutions for system of ordinary differential equation.
2. Let W(f, g) be the wronskian of two linearly independent solutions f and g of the equation $\ddot{W}+P(z)\dot{W}+Q(z)W=0$. Then find the value of product of W(f, g)P(z).

3. Show that o is the regular singular and 1 is the irregular singular points of the differential equation $(z-1)\ddot{W}(z)+(\cot \pi z)W(z)+\cos ec^2 \pi z\, W(z)=0$

4. Show that $\frac{dy}{dx}=3y^{\frac{2}{3}}, y(0)=0$ has more than one solution and indicate the possible reason.

5. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of $x^2\ddot{y}-2x\dot{y}-4y=0$,forall x in [0,10] *consider the Wronskian* $W(x)=y_1(x)\, y_2'(x)-y_1'(x)\, y_2(x)$. *If* $W(1)=1$ then find the value of $W(3)-W(2)$?

6. Find the series solution near z=0 **of** $2z^2W''(z)+zW'(z)-(z+1)W(z)=0$

Using the fact that $y=x^2$ is a solution of the ordinary differential equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=0, x>0$, find the another independent solution.

7. Solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\cos ecax.$

8.Find the solution of the differential equation $(y'''-y''+3y'-y)=e^x\cos 2x$.

9. A function satisfied the differential equation $\frac{d^2N}{dx^2}-\frac{N}{L^2}=0$ where L is constant. The boundary conditions are $N(0)=k$ and $N(\infty)=0$. Then Find the solution of the above solution.

10. Find the particular integrals of the ODE : $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=3e^{2x}$,

11. Find the steady state point of $\frac{dy}{dx}=e^{x-1}-1$ and $\frac{dy}{dx}=ye^x$

12. Discuss about the stability of the following system of differential equations; $\frac{dy}{dx}=-x+y$ and $\frac{dy}{dx}=4x-y$

13. Discuss about the stability of the following system of differential equations; $\frac{dy}{dx}=x+2y$ and $\frac{dy}{dx}=x^2+y$

14. Determine the steady state and their stability of the differential equation $\frac{dy}{dx}=f(y)=y^2-5y+6.$

15. Solve $xz^3dx-zdy+2ydz=0$

16. Solve $3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$

17. Show that (y+z)dx+(z+x)dy+(x+y)dz=0 is exact and homogeneous. Hence solve it.

18. Show that $f(x, y)=xy^2$ on $\mathbb{R}$: { |x|≤1, |y|≤1} satisfying a Lipschitz condition. But this function does not satisfy a Lipschitz condition on the strip *S*: { |x|≤1, |y|≤∞}.

19. Solve the following system of differential equations;

$$\frac{dy}{dx}+4x+3y=t\ , \qquad \frac{dy}{dx}+2x+5y=e^{t}\ .$$

20. Find the fundamental matrix and complementary solution of the homogenous linear system of differential equations : $\frac{dx}{dt}=3x+y,$ $\frac{dy}{dt}=x+3y.$

21. Solve : $(1+3x)^2\frac{d^2y}{dx^2}-6(1+3x)\frac{dy}{dx}+6y=8(1+3x)^2\ ,-\frac{1}{3}<x<\infty$

22. Find the power series solution of the equation $4x^2y''(x)+2xy'(x)-(x+4)y=0$ in power of x.

23. If $\vec{U}$, $\vec{V}$, $\vec{W}$ be three mutually perpendicular unit vectors such that $\vec{V}\times\vec{W}=\vec{U}$ then prove that $\vec{V}=\vec{W}\times\vec{U}$ & $\vec{W}=\vec{U}\times\vec{V}$

24. Given that $\vec{r}=a\cos U\ \hat{i}+a\sin U\ \hat{j}+bU\ \hat{k}$ show that $[\vec{r}\vec{r}\vec{r}]$ is independent of `U'

25. Derive the volume of a tetrahedron whose co-ordinates of vertices are given. Use it to calculate the volume of the tetrahedron whose vertices are A(2, -1, 4), B(4, 2,3), C(3, 2,-1), D(1,2,3).

26. Let r_1 and r_2 be the roots of the indicial polynomial for the equation $\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=0$

Where a, b are constents. If $r_1 \neq r_2$, then show that two independent solutions are e^{r1x}, e^{r2x} on [a, b]

Hence deduce that $y_1 = e^{-2x}$, and $y_2 = e^{-3x}$ are two independent solution of the above ODE if $a = 5 \,\&\, b = 6$

27. Apply the method of variation of parameter to solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$

28. Knowing that $y = x$ is a solution of the ODE $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = 0 (x \neq 0)$

Reduce the equation $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3 (x \neq 0)$ to a differential equation of 1^{st} order &1^{st} degree and final its complete primitive.

29.Solve

$(1+3x)^2\frac{d^2y}{dx^2} - 6(1+3x)\frac{dy}{dx} + 16y = 8(1+3x)^2, \frac{1}{3} < x <$ □

30. The wronskian of two solutions of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0, a_0$ $(x \neq 0), x$ belongs to $[a, b]$ is either

Identically zero or never zero on $[a, b]$

31. Find the fundamental and complete solution of the homogeneous system of ODE $\frac{dx}{dt} = 3x + y$ and $\frac{dy}{dt} = x + 3y$

32. Solve $\frac{dx}{dt} + 4x + 3y = t$ and $\frac{dy}{dt} + 2x + 5y = e^t$

33.Solve $(D^2+1)x+(D+1)y=t, 2x+(D+1)y=0$ with $x=y=0$, at $t=0$

34. Find the phase curve of the system of dyanamical equation$x=-x-2y$ and $y=2x-y$. Also show that the system is stable.

35. Find the solution of the $4x^2y''(x)+2xy'(x)-(x+4)y= 0$ in power of x.

36. Find the power series solution of the equation in power of x $(x^2-1)y''+4xy'+3xy=0$, given that $y(0)=5, y'(0)=7$.

37. Find the general solution of the homogeneous equation $\frac{d\vec{x}}{dt}=\begin{pmatrix}1 & -2 & 0\\ 2 & 3 & 0\\ 0 & 0 & 2\end{pmatrix}\vec{x}$

where $\vec{x}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$

38. Find the general solution of the homogeneous equation $\frac{d\vec{x}}{dt}=\begin{pmatrix}1 & -2 & 0\\ 2 & 3 & 0\\ 0 & 0 & 2\end{pmatrix}\vec{x}$

where $\vec{x}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$

39. Define fundamental set of solutions and fundamental matrix for system of differential equations.

40. Determine whether the matrix $B=\begin{pmatrix}e^{4t} & 0 & 2e^{4t}\\ 2e^{4t} & 3e^{4t} & 4e^{4t}\\ e^{4t} & e^{t} & 2e^{4t}\end{pmatrix}$

Is a fundamental matrix or not? Justify your answer.

$X=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$ $A=\begin{pmatrix}1 & -3 & 9\\ 0 & -5 & 18\\ 0 & -3 & 10\end{pmatrix}$.

41. Define fundamental set of solutions for system of ordinary differential equation.

42. What is meant by singularity of a linear ordinary differential equation?

43. Find the regular singular point of the ODE : $x(x-1)y''+(\sin x)y'+2x(x-1)y=0$.

44. Discuss Frobenious method of finding the series solution about the regular singularpoint at the origin for an ODE of 2nd order when the roots of the indicial equation are equal.

45. Find the series solution near z=0 of $(z+z^2+z^3)\ddot{W}(z)+3z^2\dot{W}(z)-2W(z)=0$
Find the general solution of the ODE $2zw''(z)+(1+z)w'(z)-kw=0$. (where k is a real constant) in series form. For which values of k is there a polynomial solution?

46. Find all the singularities of the following differential equation and then classify them: $(z-z^2)^2\omega''+(1-5z)\omega'-4\omega=0$.

47. Define a self-adjoint differential equation with an example.

48. Find all the singularities of the following differential equation and then classify:
$z^2(z^2-1)^2\omega''-z(1-z)\omega'+2\omega=0$.

49. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of $x^2\ddot{y}-2x\dot{y}-4y=0$, forall x in [0,10] $considertheWronskian$ W(x) = $y_1(x)y_2'(x)-y_1'(x)\,y_2(x)$. $If W(1)$ =1 then find the value of W(3)−$W(2)$?

50. Let $w_1(z)$ and $w_2(z)$ be two solutions of $(1-z^2)w''''(z)-2zw''(z)+(\sec z)w=0$ with Wroinskian w(z). If $w_1(0)=1$, $w''(0)=0$, and w $(\frac{1}{2})$ $=\frac{1}{3}$, then find the value of $w_2'(z)$ at z=0.

51. Prove that $\left[(\vec{\alpha}\times\vec{\beta})(\vec{\beta}\times\vec{\gamma})(\vec{\gamma}\times\vec{\alpha})\right]=[\vec{\alpha}\vec{\beta}\vec{\gamma}]^2$.

52. State the lipschitz condition for function f(x,y) on D

53. State the picard theorem for existence and uniqueness for $\frac{dy}{dx}=f(x,y)$ with y(x)=y_0. Also show for $\frac{dy}{dx}=\frac{1}{y}, y(0)=0$, has more than one solution indicate the possible reason.

54. If $y(x) = x^2 \sin x$ is a solution of an n-th order linear ODE $y^n(x) + a_1 y^{(n-1)}(x) + \dots\dots\dots + a_{(n-1)} y(x) + a_n y(x) = 0$ with real constant coefficients, then prove that the least possible value of n is 6.

55. Show that the solution of the ODE $\frac{dx}{dt} = 2x + y$ & $\frac{dy}{dt} = 3x$ satisfy the relation $3x + y = Ke^{3t}$.where K is a real constant.
56. Find the equilibrium point for the system of equations $\dot{x} = 2x + 7y$ and $\dot{y} = x^2 + 3y$.
57. Find the ordinary &singular point of the ODE $3x^2 y'' + 7x(x + 2)y' - 4y = 0$.
58. Show that $x = 0$ is a ordinary point and $x = 2$ is a regular singular point of the ODE $x(x - 2)y'' + (sinx)y' + 2x(x - 2)y = 0$.
59. Find the P. I of the ODE $(D^4 + 2D^2 - 4)y = e^{2x}$.
60. Show that the general solution of the ODE with constant coefficient $y'' + by' + cy = 0$ approaches to zero as $x \to \infty$ if both *b* & *c* are positive.
61. Show that the vector type equation here $F = (2x - yz)\hat{\imath} + (2y - zx)\hat{\jmath} + (2z - xy)\hat{k}$ is irrational.
62. Prove that the $\vec{\nabla}^2 f(x) = \frac{d^2t}{dr^2} + \frac{2}{r}\frac{dt}{dr}$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

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